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DYNAMIC REVENUE GUARANTEES IN PUBLIC PRIVATE PARTNERSHIPS: A PRIMAL AND DUAL VALUATION APPROACH

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Abstract

Public Private Partnerships (PPP) contracts bring together the public and the private sectors in order to provide public service and to develop public infrastructure. If implemented appropriately, these contracts may modernize the public procurement and help governmental agencies overcome the budgetary constraints. One of the key success factors of PPP contracts is the appropriate risk sharing between the public and the private sectors. The Minimum Revenue Guarantee (MRG) is a well spread solution for revenue risk mitigation. The government grants the private sector a minimum revenue that ensures the project profitability. The MRG is a real option with the possibility of multiple exercises before the end of the project. This paper presents a methodology for determining a lower and an upper bounds value for dynamic minimum revenue guarantee contracts where the exercise dates should at least be separate by a certain refraction period.

Key words

Public Private Partnerships, Concession, Minimum Revenue Guarantee, Refraction period, Real options, Risk sharing, Multiple exercise option, Information relaxation, dual valuation

Introduction

The development of Public Private Partnerships has helped governmental entities to fund new projects in order to fulfill the increasing demand for public service provision. The PPP scheme has known a tremendous success in both developed and developing countries. Its application has made a huge contribution to the development of infrastructure worldwide. The system provides an effective route to mobilize private funds and business expertise for the development of public infrastructure.

PPP are generally conducted under a project finance framework, where a Special Purpose Vehicle (SPV) is in charge of the project's funding, constructing and maintaining for a certain period of time. Afterwards, the project is transferred back to the public entity. The SPV is funded by both equity and non-recourse debt with a high ratio of debt to capital ranging commonly from 70% to 95%. The project success under project finance schemes depends heavily on the project's ability to generate sufficient revenue to operate and maintain the structure, to serve debt and to remunerate equity. Unpredictability of future demand, the irreversibility of the investment combined with the long term commitment make such projects very risky. Public entities have, therefore, to provide enough incentive for private bidders to make the project appealing and to ensure that they are able to recoup a reasonable return on their investment.

Revenue risk is one of the most significant risks in PPPs. It may lead to huge consequences on the project company. The cash flow arising from the project may turn out insufficient to ensure its financial viability. In that case, the public service provision may be considerably affected and in some extreme cases shut down. The revenue risk has, generally, no impact on the governmental entity budget. In contrast, the political impact may be substantial, if end users are directly affected by a low-quality or an interrupted service. Imagine for a while the whole water provision system in a city shut down.

Under such circumstances, the government has to be cooperative and public deciders are generally willing to bail out the project's company in order to avoid political consequences and to assure the continuity of the public service provision. The PPP scheme is, actually, a relationship specific investment as it was pointed out by (Dong and Chiara, 2010). In other words, the PPP contract is a cooperative game where the interest of both players lies in positive interaction with variable rewards that depend on each party bargaining power and her willingness to make concessions.

The bail out process is, generally, done via the renegotiation of the contract initial terms which requires a lot of resources and leads to high transactional costs. Incorporating risk mitigation mechanisms in the initial contract should help allocating these lost resources to other public projects. The most known solution among practitioners and academics for demand risk sharing in PPPs is the Minimum Revenue Guarantee contract. Under such commitments, the public entity secures a certain minimum revenue to maintain the project's financial viability. This guarantee may be seen as a real-option that the public entity offers to the SPV free of charges. The option being free, does not mean, in any case, that it does not add value to the public entity. Its cost may be in fact, counterbalanced by the social benefit that the continuation of the public service provision leads to.

Once a revenue guarantee is implemented within the PPP contract, it increases the project's value and enhances, consequently, its appeal for private bidders. (Brandao and Saraiva, 2008) mentioned, for instance, that the Costenera Norte toll in Chile had no bidders when it was first auctioned in 1998. Only in 2000, after the government support was included, the road was successfully bid. The importance of the MRGs should not decrease the vigilance of the public entities and should not lead to unreasonable practices. (Irwin, 2007) reported that some governments did not even account for such guarantees. The MRG contract nature as a risk mitigation mechanism should be dealt with very carefully since it may bring several future liabilities and huge fiscal burdens for tax-payers. Proper evaluation tools are, therefore, needed to develop appropriate provision and accounting mechanisms.

The valuation of Minimum Revenue Guarantees has been an active subject of research during the last years. The main trend of researchers, among them (Dailami et al., 1999; Brandao and Saraiva, 2008; Ashuri et al., 2011; Iyer and Sagheer, 2011; Cheah and Liu, 2006), has structured the contract in a "static" manner: the private partner has to fix, prior to the beginning of the contract, her potential exercise dates (the dates when she will redeem her guarantee). Under this setting, the contract is similar to a portfolio of European options maturing at each chosen exercise date. This structure makes the contract very hard to manage because of the difficulty to select adequate exercise dates before the contract's signing. To remedy this inefficiency, a high number of exercise rights should be offered to the private partner which increases the option cost and the public exposure to risk.

Another trend, that this work follows, was initiated by (Chiara et al., 2007). The guarantee contract may leave the freedom to the decision maker to select her exercise dates during the project's lifetime. The private entity can then take advantage from the information revealed over time. This feature makes the contract more flexible. It allows, moreover, to design the guarantee with a lower number of exercise rights which limits the exposure of the public entity to demand risk.

The American contract is more suitable for the private entity, its cost is, however, higher to the public entity. This work proposes a novel contract that presents a certain trade-off between flexibility and cost. For this purpose, the American-style contract is generalized by constraining the dates at which the guarantee can be redeemed. Under the proposed framework, two consecutive exercise rights has to be at least distant by a certain period δ referred to as a refraction period . Its introduction allows to come up with more innovative approaches during the contract design. The provision for the guarantee can be, moreover, spread over several years. This feature should, then, lead to an easier budgetary management and provides a better visibility for the guarantee's cash flows. The MRG with a refraction period lies somewhere between the European contract and the American one and should help both parties reaching "win-win" agreements.

Inspired by the work on American and swing options (Rogers, 2002; Haugh and Kogan, 2004; Andersen and Broadie, 2004; Meinshausen and Hambly, 2004; Schoenmakers, 2012, 2009; Bender, 2011; Bender et al., 2013), this work adopts a primal and a dual valuation approach. The proposed methodology aims to derive reliable bounds on the contract value. The primal approach determines a lower bound on the guarantee value using Monte Carlo Simulation and regression technique introduced by (Longstaff and Schwartz, 2001) as it was suggested in (Chiara et al., 2007). Starting from the results of the primal problem, the dual approach determines an upper bound on the option value. It is based on information relaxation introduced in (Brown et al., 2010) and relies on Martingale inequalities introduced in (Bender, 2011; Bender et al., 2013) for swing options with refraction periods. The use of the dual approach is essential for Minimum Revenue Guarantees valuation since it allows to assess the quality of the determined value especially in the absence of a benchmark because of the scarcity of market data.

The remainder of this paper unfolds as follows. A model for the guarantee contract is presented in section 1. Section 2 tackles the contract's valuation. The methodology is then applied, in section 3, to a PPP project. A conclusion is, finally, drawn in 4.

1 The dynamic MRG contract model

Under a Public Private Partnership contract, a public entity entrusts a third party (a private entity) with the building, maintaining and operating of a public infrastructure. Generally a Special Purpose Vehicle is established in order to conduct all the operations related to the project. There are several forms for the SPV to be rewarded for her services. The most common one is the right to collect tolls during a certain period of time. A PPP contract with the right to charge tolls is often referred to as a concession agreement. The duration of the contract T is known as the concession duration. The revenue that arises from the project is not known before the beginning of the concession and may be insufficient to maintain the financial viability which threatens the project success and the public service provision. For low-demand projects, the public entity may offer some guarantees. This development focuses on Minimum Revenue Guarantees. Such contracts grants the SPV the revenue shortfalls occurring at certain years of the concession. For this purpose, the two parties agree upon a certain minimum revenue guaranteed K_t for each year until the end of the operation phase $t \in \{1, \cdots, T\}$. The project's revenue is uncertain and follows a Markovian stochastic process $\{R_t\}_{t=0}^{T}$ which evolves in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. If the project's revenue falls below the threshold K_t for a certain year, the SPV has the right and not the obligation to claim a compensation equal to the revenue shortfall $Z_t = \max(0, K_t - R_t)$. The compensation structure is similar to a financial put option on the underlying asset R_t with an exercise strike of K_t. The governmental agency may offer the SPV a full coverage contract, under which she can claim all the revenue shortfalls during the operation phase. In other cases, the contract may offer a limited number of exercise rights $n < T$. Let $(\tau_1, \cdots, \tau_n)$ be the dates at which the guarantee is claimed. They can be either fixed before the beginning of the contract in an European style, or chosen during the contract's life in an American style. Under the European contract, the dates are assumed, without loss of generality, to be chosen at the beginning of the concession [1] (i.e $\tau_i = i, \quad i = 1, \cdots, n$) . The value of the contract V^{static} is then equal to the value of a portfolio of n European put options maturing at $\tau_1, \cdots, \tau_n$ with respective exercise strikes of $K_{\tau_1}, \cdots, K_{\tau_n}$:

$$V_t^{\text{static}}(n) = \mathbb{E}_t \left[\sum_{i=1}^{n} Z_{\tau_i} \right], \tag{1}$$

[1]This assumption is motivated by the fact that the uncertainty is higher at the beginning of the contract, although this may still arguable.

where $\mathbb{E}_t$ denotes the conditional expectation based on the historical observations up to t: $\mathbb{E}_t[.] = \mathbb{E}[.|\mathcal{F}_t]$.

Under the American contract, the SPV has the freedom to choose the exercise dates during the contract life. She can, hence, adapt her exercise dates according to the realizations of the project's revenue. Under this setting, the SPV manages the contract via a set of stopping time $(\tau_1, \cdots, \tau_n)$ that is referred to as an exercise policy. This policy is unknown at the beginning of the contract and depends on the realizations of the revenue (i.e for the realization of two different scenarios, the SPV can select two different sets of exercise dates. Under the European contract, however, the dates are always fixed). This policy is determined in a non-anticipative manner since the private partner is not able to foresee the future. Under the proposed framework, two consecutive exercise dates should be at least distant by a refraction period $\delta \in \mathbb{N}$. The maximal number of exercise rights that the SPV can be benefit from is $n_{\max}(\delta) = \lfloor \frac{T}{\delta} \rfloor . \lfloor x \rfloor$ denotes the floor function of the real valued variable x. The set of admissible exercise policies $\Pi_t(n, \delta)$ at time t is given by :

$$\Pi_t(n, \delta) = \big\{ (\tau_1, \cdots, \tau_n) \in [t, T] \cup \{+\infty\} : \tau_i \geq \tau_{i+1} + \delta, i = 1, \cdots, n \big\}. \qquad (2)$$

Some exercise rights can go unused, one can assume that they are exercised at $+\infty$ and sets $Z_{+\infty} \equiv 0$. The contract guarantee fair value $V_t(n, \delta)$ is the maximal expected compensations obtainable by the private partner over the life of the project through non-anticipative admissible policies:

$$V_t(n, \delta) = \sup_{\pi_t(n,\delta) \in \Pi_t(n,\delta)} \mathbb{E}_t \left[\sum_{k=1}^{n} Z^d_{\tau_k(t,n)} \right], \qquad (3)$$

where $\pi_t(n, \delta)$ refers to an exercise policy, $Z^d_{\tau_k(t,n)} = Z_{\tau_k(t,n)} \frac{B_t}{B_{\tau_k(t,n)}}$ is the discounted compensation to time t by the means of a bank account $\{B_i\}_{i=0}^{T}$. For the contract value to exist, the compensation Z_t has to satisfy: $\mathbb{E}[\max_t |Z_t|] < +\infty$. For $n > n_{\max}(\delta)$, one has $V_t(n, \delta) = V_t(n_{\max}(\delta), \delta)$.

If all the allowed exercise rights are exercised prior to t , the decision maker has no choice but continuing until the end of the concession. Otherwise, she chooses among two decisions:

- Claim a compensation Z_t and hold a contract with $n-1$ rights where she can exercise her $(n-1)^{\text{th}}$ right at least after a period δ,
- Continue without exercising and hold the same contract where she can exercise her n^{th}

right starting from the next period.

Let Q^n_{t+1} be the continuation value one step later for the option with n rights. It measures the expected reward if no exercise is made. Similarly, one introduces the continuation value $Q^{n-1}_{t+\delta}$ for the option with one exercise right less at δ steps later:

$$\begin{cases} Q^n_{t+1} = \mathbb{E}_t \left[\frac{B_t}{B_{t+1}} V_{t+1}(n, \delta) \right], \\ Q^{n-1}_{t+\delta} = \mathbb{E}_t \left[\frac{B_t}{B_{t+\delta}} V_{t+\delta}(n-1, \delta) \right]. \end{cases} \tag{4}$$

The decision maker's behavior can be modeled via an exercise indicator $I_t(n, \delta)$ as follows:

$$I_t(n, \delta) = \begin{cases} 0 & \text{if} \quad Z_t + Q^{n-1}_{t+\delta} < Q^n_{t+1}, \\ 1 & \text{otherwise}, \end{cases} \tag{5}$$

where 0 stands for continuation and 1 for exercise.

2 Valuation of the MRG contract

2.1 The primal problem: determining a good exercise policy

The optimal exercise policy cannot be reached in general, however, one can determine a good combination of stopping times that approximates the real-option value. The contract valuation can be done by the means of dynamic programming. The idea is to model the decision process recursively. At the expiration date, the contract value is equal to the terminal compensation: $V_T = Z_T$. From this point, one can go backward in time using the following Bellman equation:

$$\begin{cases} V_T = Z_T, \\ V_t = \max\left(Z_t + Q^{n-1}_{t+\delta}, Q^n_{t+1}\right). \end{cases} \tag{6}$$

The computation of the two continuation values within a Monte Carlo simulation can be cumbersome. Thanks to the seminal work of (Longstaff and Schwartz, 2001; Tsitsiklis and Van Roy, 2001; Carriere, 1996), this computational burden can be alleviated. This work relies on the methodology of (Longstaff and Schwartz, 2001), where the continuation values are approximated by the means of a regression on the current state space. In other words, future information can be transformed to present information by the means of a regression that models the expertise and knowledge of the decision maker. An approximation $\tilde{Q}^n_{t+1}$ (resp. $\tilde{Q}^{n-1}_{t+\delta}$) of Q^n_{t+1} (resp. $Q^{n-1}_{t+\delta}$) is then obtained as follows:

$$\begin{cases} \tilde{Q}^n_{t+1} = \sum_{r=1}^{N_b} \alpha^n_{r,t} \Psi_{r,t}(R_t), \\ \tilde{Q}^{n-1}_{t+\delta} = \sum_{r=1}^{N_b} \beta^{n-1}_{r,t} \Psi_{r,t}(R_t), \end{cases} \tag{7}$$

where $\{\Psi_{r,t}(.)\}_{r=1}^{N_b}$ is a set of N_b basis functions. The coefficients $\left\{\alpha^n_{r,t}\right\}_{r=1}^{N_b}$ and $\left\{\beta^{n-1}_{r,t}\right\}_{r=1}^{N_b}$ are determined by least squares regressions:

$$\begin{cases} \left\{\alpha^n_{r,t}\right\}_{r=1}^{N_b} = \text{argmin} \left\{ \sum_{h=1}^{N_s} \left|\left|Q^n_{t+1,h} - \tilde{Q}^n_{t+1,h}\right|\right|^2 \right\}, \\ \left\{\beta^{n-1}_{r,t}\right\}_{r=1}^{N_b} = \text{argmin} \left\{ \sum_{h=1}^{N_s} \left|\left|Q^{n-1}_{t+\delta,h} - \tilde{Q}^{n-1}_{t+\delta,h}\right|\right|^2 \right\}, \end{cases} \tag{8}$$

where N_s is the number of Monte Carlo simulations. A lower bound value $\underline{V}_t\,(n,\delta)$ and an approximate exercise indicator $\tilde{I}_t\,(n,\delta)$ can be obtained replacing the continuation values by their approximations in (5) and (6) . A near optimal exercise policy can be determined as follows:

$$\begin{cases} \tilde{\tau}_n = \inf\left\{t | \tilde{I}_t\,(n,\delta) = 1\right\}, \\ \tilde{\tau}_k = \inf\left\{t \geq \tilde{\tau}_{k+1} + \delta, |\tilde{I}_t\,(k,\delta) = 1\right\}, k = n-1, \cdots, 1. \end{cases} \tag{9}$$

In order to avoid any anticipative bias that may arise from the use of future information in the regression step, it is preferable to use two realizations of the revenue process $\{R_t\}_{t=0}^T$. The first set is used to determine the regression coefficients which are applied to the second set to model the decision maker's behavior (Glasserman, 2003).

2.2 The dual problem: determining a good upper bound to the contract value

Relaxing the non-anticipativity constraint imposed in the primal problem leads to an upper bound on the contract value. A natural relaxation is to allow the decision maker to foresee all the future. In that case, an upper bound can be determined as follows:

$$V_t(n,\delta) \leq \mathbb{E}_t \left[\max_{\pi_t(n,\delta) \in \Pi_t(n,\delta)} \sum_{k=1}^{n} Z^d_{\tau_k(t,n)} \right]. \tag{10}$$

In general, this bound is of a poor quality. However, it can be improved by penalizing the use of future information in the optimization process (Brown et al., 2010). This work relies on Martingale penalties introduced in (Bender, 2011; Bender et al., 2013). These penalties are determined via the Doob-Meyer decomposition of the contract value. An improved upper bound can be determined as follows:

$$\begin{aligned} V_t(n,\delta) \leq & \mathbb{E}_t\Bigg[\sup_{(t_1,\ldots,t_n)\in\Pi_t(n,\delta)}\Bigg\{\sum_{k=1}^{n-1} Z^d_{t_k} - M^k_{t_k} + M^k_{t_{k+1}} + A^k_{t_{k+1}+\delta} \\ & - \mathbb{E}_{t_{k+1}}\left[A^k_{t_{k+1}+\delta}\right] + Z_{t_n} - M^n_{t_n} + M^n_t\Bigg\}\Bigg], \end{aligned} \tag{11}$$

where $\left\{M^k_t\right\}_{k=1}^n$ and $\left\{A^k_t\right\}_{k=1}^n$ are respectively a family of martingales and integrable predictable processes starting from 0 at $t = 0$. Furthermore, if $\left\{M^{*k}_t\right\}_{k=1}^n$ and $\left\{A^{*k}_t\right\}_{k=1}^n$ are obtained via the Doob-Meyer decomposition of the Snell envelopes $\{V_t(k,\delta)\}_{k=1}^n$, then the upper bound matches with the contract optimal value. For $\delta = 1$, the predictability of the processes leads to the dual formulation of (Schoenmakers, 2009, 2012):

$$V_t(n,1) \leq \mathbb{E}_t\left[\sup_{t_1,\ldots,t_n)\in\Pi_t(n,1)}\left\{\sum_{k=1}^{n} Z^d_{t_k} - M^k_{t_k} + M^k_{t_{k+1}}\right\}\right], \tag{12}$$

with $t_{k+1} \equiv t$. Another dual formulation for the case $\delta = 1$ can be found in (Meinshausen and Hambly, 2004) for the marginal value $\Delta V_t(n) = V_t(n+1,1) - V(n,1)$:

$$\Delta V_t(n) = \inf_{\pi_t(n-1)}\inf_{M_j\in\mathbb{M}_0}\left\{\mathbb{E}_t\Big[\max_{j\in\mathbb{T}\setminus\{\tau_{n-1},\ldots,\tau_1\}}(Z^d_j - M^n_j + M^n_t)\Big]\right\}, \tag{13}$$

where $\pi_t(n-1)$ is an exercise policy for the contract with $n-1$ exercise rights and $\mathbb{T} = \{t,\cdots,T\}$. The infimum is reached for the optimal stopping policy and the Martingale resulting from the Doob-Meyer decomposition of the marginal value.

The optimization procedure in (11) can be very time consuming especially in the case of a high number of exercise rights and long maturities. The problem structure allows to conduct the optimization recursively. One can introduce for this purpose:

$$\begin{aligned} \lambda^{i,n,\delta}_t = & \max_{j_0=t\leq j_1\leq j_2+\delta\leq\ldots\leq j_{n-i}+\delta}\sum_{k=1}^{n-i} Z^d_{j_k} + M^{n-i-k+1}_{j_{k-1}} - M^{n-i-k+1}_{j_k} \\ & + \mathbb{I}_{k>1}\left(A^{n-i-k+1}_{t_{j_{k-1}}+\delta} - \mathbb{E}_{t_{j_{k-1}}} A^{n-i-k+1}_{t_{j_{k-1}}+\delta}\right), \end{aligned} \tag{14}$$

and one has by construction:

$$V_t(n,\delta) \leq \mathbb{E}\left[\lambda_t^{0,n,\delta}\right]. \tag{15}$$

$\lambda_t^{i,n,\delta}$ can be determined recursively as follows:

$$\begin{aligned}\lambda_t^{i,n,\delta} = \max\Big\{&\lambda_{t+1}^{i,n} - M_{t+1}^{n-i} + M_t^{n-i}, \\ &Z_t^d + \lambda_{t+\delta}^{i+1,n} - M_{t+\delta}^{n-i-1} + M_t^{n-i-1} + A_{t+\delta}^{n-i-1} - \mathbb{E}_t A_{t+\delta}^{n-i-1}\Big\}.\end{aligned} \tag{16}$$

A detailed proof for this simplification for a more general case can be found in (Bender et al., 2013). The determination of the optimal Martingales and predictable processes is as hard as the determination of the optimal policy, but an approximation can be built from the lower bound value and this leads to an upper bound $\overline{V}_t(n,\delta)$ on the contract value. The algorithms used in this work are the same as the ones presented in the original papers.

Once the upper and lower bounds are computed, one can construct a confidence interval at $(1-\alpha)\%$ for the option price:

$$\left[\underline{V}_0(n,\delta) - \beta\frac{\underline{\sigma}_n}{\sqrt{N_l}}, \qquad \overline{V}_0(n,\delta) + \beta\frac{\overline{\sigma}_n}{\sqrt{N_u}}\right], \tag{17}$$

where $\underline{\sigma}_n$ (resp. $\overline{\sigma}_n$) is the volatility of the lower (resp. upper) bound estimate, N_l(resp N_u) is the number of simulations used for the computation of the lower (resp. upper) bound and $\beta = \phi^{-1}(1-\frac{\alpha}{2})$ with ϕ^{-1} is the inverse cumulative distribution function of the standard normal distribution.

3 Numerical experiments

This section aims to illustrate the described methodology via a PPP project. The construction duration is estimated at 3 years for a total cost of 300 Million € (present value). The construction costs inflation is modeled by a symmetric triangular distribution between 1% and 3%. The SPV is funded by equity (15%) and non-recourse debt (85%). The lenders grant the SPV a grace period during the construction of the project where she does not reimburse capital and pays only an interest of 3%. Once the construction is over, the interest on the borrowed capital is of 5.5%. The debt's maturity is of 25 years starting from the construction termination. The expected return on equity is determined using the Capital Asset Pricing Model and is of 10% [2]. The SPV is allowed to collect tolls during 30 years of the 40 years of the project's life. After the construction is over, the project requires an annual cost of 2 Million € for operation and maintenance. The inflation of this cost is modeled by a symmetric triangular distribution between 1% and 2%. The tax rate is 33%. The project's revenue is assumed to follow a Geometric Brownian motion:

$$dR_t = \mu R_t dt + \sigma R_t dW_t, \tag{18}$$

where μ =1% is the annual expected revenue increment, σ =7% is the revenue volatility and W_t is a Wiener process. The initial value of the revenue is estimated at 20 Million €.
The project's revenue is not sufficient to maintain its financial viability. The public entity is, therefore, willing to provide a minimum revenue guarantee during certain years of the concession. The guaranteed revenue is given by $K_0 = 19$ and $K_{t+1} = 1.02K_t$.

3.1 *The contract value*

First lets determine the MRG contract "intrinsic" value from the private partner perspective. This process allows to validate the use of the regression approach and to assess the gap between the lower and upper bounds on the contract's value. The numerical experiment is based on 50 000 Monte Carlo simulations to determine the regression coefficients and to compute the lower bound , 50 simulations for the upper bound calculation and 200 simulations for the nested Monte Carlo simulations required for the approximations of the Martingales and the integrable processes (Meinshausen and Hambly, 2004; Schoenmakers, 2012, 2009; Bender et al., 2013). The regression is conducted using the first 7 Laguerre polynomials, as well as the payoff functions. Figure 1 provides a comparison of the European contract with the flexible contracts with refraction periods of $\delta = 1, 2, 3$. One can see that the introduction of the refraction period reduces the flexible contract value. The superiority towards the European contract is still maintained in the region of feasible number of exercise rights[3]. Table 1 presents the lower and upper value for the MRG contract with $\delta = 1$. It permits also to assess the quality of the dual formulations of (Meinshausen and Hambly, 2004) and (Schoenmakers, 2012) in the case

[2]Here we work under the real world measure and assume that the project's risk is similar to the company's risk. The risk premium is derived from the CAPM. The valuation provided is non consistent with market, however the approach permits to assess the financial viability of the project in the real world and to take adequate measures to manage the project's risk. Under the risk neutral measure, such approach is not possible due to the deformation of the stopping times which changes the financial indicators of the project and should lead to erroneous decisions.

[3]$\{n \in \mathbb{N} \quad |n \leq n_{\max}(\delta)\}$

of MRG contracts. The numerical experiment shows that the latter approach outperforms the former. In fact, in (Schoenmakers, 2012) the optimization is over the set of approximated martingales and is conducted in one step. However, in (Meinshausen and Hambly, 2004), there are n optimizations done simultaneously on approximated martingales and approximated stopping times which increases the upper bias of the contract value. Table 2 presents the lower and upper bounds for the contracts with refraction periods: $\delta = 2, 3, 4$. One can see that the dual and upper bound are always tight, and the set of parameters used for the regression can be validated.

n	$\underline{V}_0$	$\overline{V}_0$ Meinshausen & Hambly	$\overline{V}_0$ Schoenmakers	C.I at 99% Schoenmakers
1	1.378	1.378	1.378	[1.374 , 1.379]
2	2.726	2.745	2.728	[2.718 , 2.731]
3	4.037	4.087	4.043	[4.026 , 4.050]
4	5.311	5.388	5.321	[5.296 , 5.330]
5	6.545	6.651	6.558	[6.526 , 6.571]
6	7.738	7.871	7.760	[7.716 , 7.775]
7	8.890	9.050	8.914	[8.864 , 8.932]
8	9.998	10.186	10.028	[9.969 , 10.046]
9	11.064	11.280	11.091	[11.032 , 11.114]
10	12.085	12.329	12.142	[12.050 , 12.172]
11	13.061	13.336	13.093	[13.023 , 13.121]
12	13.993	14.300	14.037	[13.952 , 14.074]
13	14.879	15.218	14.934	[14.836 , 14.969]
14	15.721	16.093	15.786	[15.675 , 15.825]
15	16.518	16.925	16.608	[16.469 , 16.656]
16	17.271	17.718	17.391	[17.220 , 17.451]
17	17.981	18.463	18.103	[17.928 , 18.171]
18	18.649	19.171	18.782	[18.593 , 18.854]
19	19.273	19.844	19.436	[19.215 , 19.520]
20	19.856	20.480	20.038	[19.796 , 20.126]
21	20.395	21.073	20.582	[20.333 , 20.666]
22	20.892	21.630	21.094	[20.828 , 21.184]
23	21.345	22.153	21.530	[21.279 , 21.617]
24	21.755	22.639	21.953	[21.687 , 22.049]
25	22.120	23.085	22.355	[22.050 , 22.448]
26	22.438	23.495	22.641	[22.366 , 22.737]
27	22.706	23.862	22.907	[22.633 , 23.007]
28	22.920	24.186	23.130	[22.845 , 23.229]
29	23.073	24.453	23.277	[22.997 , 23.376]

Table 1: comparing the dual bounds of Meinshausen and Hambly and Shcoenmakers

n	$\delta = 2$		$\delta = 3$		$\delta = 4$	
	$\underline{V}_0$	$\overline{V}_0$	$\underline{V}_0$	$\overline{V}_0$	$\underline{V}_0$	$\overline{V}_0$
2	2.684	2.715	2.641	2.672	2.598	2.622
3	3.912	3.948	3.783	3.794	3.645	3.671
4	5.056	5.083	4.793	4.820	4.511	4.590
5	6.111	6.155	5.669	5.702	5.209	5.335
6	7.077	7.120	6.416	6.487	5.735	5.996

Table 2: Lower and upper bounds of MRG contracts value with different refraction periods

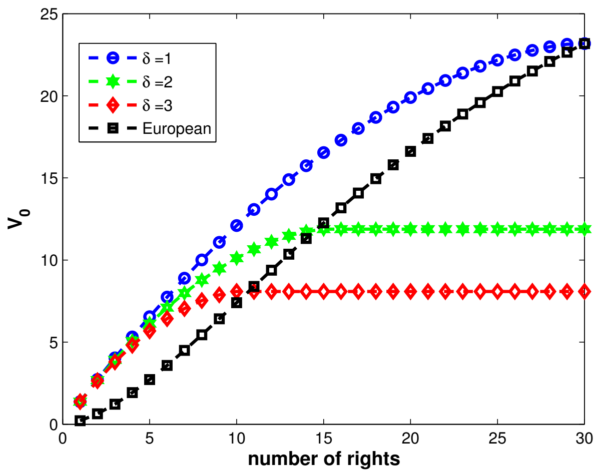

Figure 1: comparing the contract value for different refraction periods

3.2 MRG's impact on the financial viability of the project

This part examines the MRG impact on the project's financial viability. For this purpose, lets introduce the following financial indicators which are commonly used in PPP projects:

- $\mathbb{E}[NPV^s]$: the expected net present value of the project's sponsors. It is governed by the dividends that the SPV generates after she meets all her legal obligations,

- $\mathbb{E}[NPV^g]$: the expected net present value of the public entity. It is mainly governed by the guarantees that she provides and her cash flows once the project is transferred. The cash flows are discounted at the risk-free rate r_g =4%. The $\mathbb{E}[NPV^g]$ does not account for the social benefit and public deciders may be willing to accept a negative NPV^g,

- $\mathbb{E}[\mathbb{E}[DSCR_t]$: the expected value of the average Debt Service Coverage Ratio over the debt's life. The project's banakability is an increasing function of $\mathbb{E}[\mathbb{E}[DSCR_t]$. The project has at least to fulfill the following constraint to be accepted by lenders:

$\mathbb{E}[\mathbb{E}[DSCR_t] \geq 1$ [4].

For a detailed analysis on discounted cash flow analysis in Public Private Partnerships, interested readers can refer to (Zhang, 2005; Bakatjan et al., 2005; Ranasinghe, 1996).

Figure 2 illustrates the impact of the MRG on the NPV^s. It shows the enhancement of the project's return as the number of exercise rights n grows. In figure 3, one can see the opposite effect on the NPV^g. The MRG contract allows also to boost the project's bankabilty as exposed in figure 4. The profitability constraint $\mathbb{E}[NPV^s] = 0$ divides figure 2 into two distinct

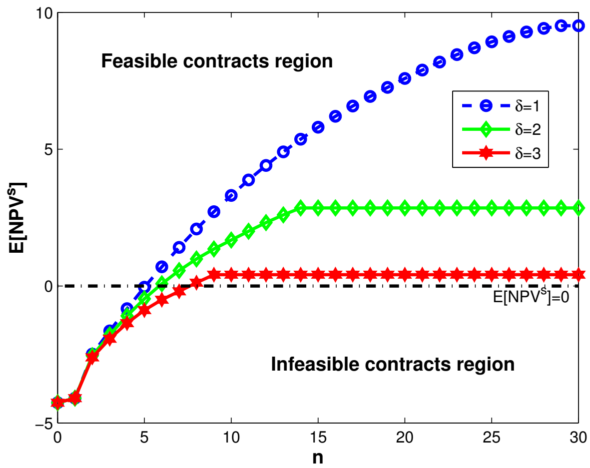

Figure 2: Comparing the impact of the MRG contract with different numbers of exercise rights and different refraction periods on the NPV^s.

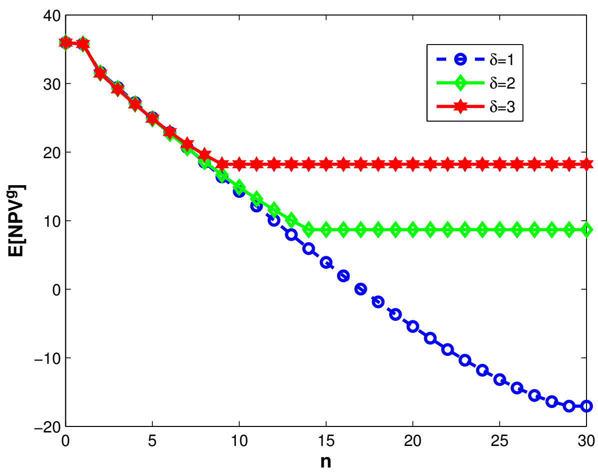

Figure 3: Comparing the impact of the MRG contract with different numbers of exercise rights and different refraction periods on the NPV^g.

[4]A higher minimal value can be imposed see (Zhang, 2005).

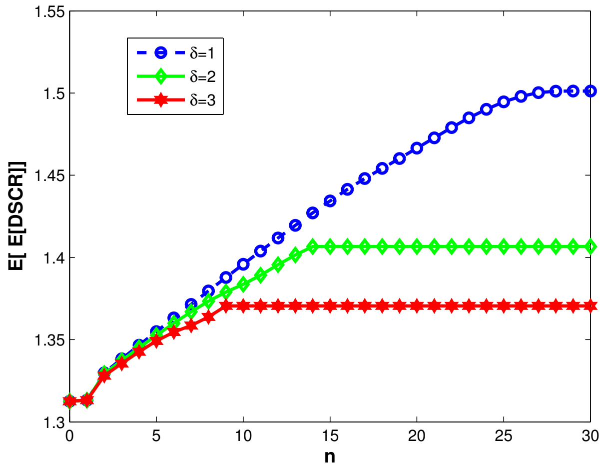

Figure 4: Comparing the impact of the MRG contract with different numbers of exercise rights and different refraction periods on the $\min_t DSCR$.

regions. The upper one contains the feasible contracts. The government can accept contracts for which $\mathbb{E}[NPV^g] < 0$ since the social benefit is not accounted for [5]. One can see that the "near" optimal[6] contract from the governmental entity's perspective can be structured with 3 different sets: $(n_1 = 7, \delta_1 = 1)$, $(n_2 = 8, \delta_2 = 2)$, $(n_3 = 9, \delta_3 = 3)$. The government faces then a trade-off between the number of exercise rights that she wants to offer and a better budgetary visibility that a higher refraction period guarantees. The final choice results from negotiation between the two parties. In this regard, the literature on the optimal design of the contract and the optimal risk sharing in PPP contracts is still scarce. The design can be seen as a bi-objective optimization procedure that yields a set of optimal contracts. Among them, the final contract is determined based on negotiation.

4 Conclusion

Minimum revenue guarantees are effective for risk sharing between the public and the private sectors in Public Private Partnerships contracts. This work extends the dynamic MRG introduced in (Chiara et al., 2007) by incorporating a refraction period that separates two consecutive exercise dates. The valuation of the guarantee was conducted in two steps. First, a primal valuation determines a near optimal exercise policy. Second, a dual valuation derives an upper bound

[5]The public tolerance for a negative net present value remains a political decision and depends on the perception of the project by the pubic decider

[6]The optimal contract is the solution of the maximization of the public net present value with respect to the constraint of private profitability. Note that the minimum revenue guaranteed can be included in the optimization program. Here, it is kept constant

on the contract's value. The proposed methodology was illustrated on a PPP project. The numerical experiment shows that the dual-approach is quite effective and leads to tight confidence interval on the guarantee value. It illustrates, furthermore, the effect of the refraction period on the project's financial viability. The introduction of the refraction period reduces the cost of the guarantee while maintaining a certain flexibility on the choice of the exercise dates. This should help design effective risk sharing contracts. The proposed framework is still applicable if the MRG contract is combined with a cap on the excess reward since the two contingent claims are mutually exclusive.

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References

L. Andersen and M. Broadie. Primal-dual simulation algorithm for pricing multidimensional american options. *Management Science*, 50(9):1222–1234, 2004.

B. Ashuri, H. Kashani, K. Molenaar, S. Lee, and J. Lu. Risk-neutral pricing approach for evaluating bot highway projects with government minimum revenue guarantee options. *Journal of Construction Engineering and Management*, 2011.

S. Bakatjan, M. Arikan, and R. L. Tiong. Closure to "optimal capital structure model for bot power projects in turkey" by sandalkhan bakatjan, metin arikan, and robert lk tiong. *Journal of Construction Engineering and Management*, 131(3):386–387, 2005.

S. Balder, A. Mahayni, and J. Schoenmakers. Primal–dual linear monte carlo algorithm for multiple stopping—an application to flexible caps. *Quantitative Finance*, 13(7):1003–1013, 2013.

C. Bender. Primal and dual pricing of multiple exercise options in continuous time. *SIAM Journal of Financial Mathematics*, 2(1):562–586, 2011.

C. Bender, J. Schoenmakers, and J. Zhang. Dual representations for general multiple stopping problems. *Mathematical Finance*, 2013.

L. E. T. Brandao and E. Saraiva. The option value of government guarantees in infrastructure projects. *Construction Management and Economics*, 26(11):1171–1180, 2008.

R. A. Brealey. *Principles of corporate finance*. Tata McGraw-Hill Education, 2012.

D. B. Brown, J. E. Smith, and P. Sun. Information relaxations and duality in stochastic dynamic programs. *Operations research*, 58(4-part-1):785–801, 2010.

J. F. Carriere. Valuation of the early-exercise price for options using simulations and nonparametric regression. *Insurance: mathematics and Economics*, 19(1):19–30, 1996.

S. S. Chandramouli and M. B. Haugh. A unified approach to multiple stopping and duality. *Operations Research Letters*, 40(4):258–264, 2012.

C. Y. Cheah and J. Liu. Valuing governmental support in infrastructure projects as real options using monte carlo simulation. *Construction Management and Economics*, 24(5):545–554, 2006.

N. Chiara, M. J. Garvin, and J. Vecer. Valuing simple multiple-exercise real options in infrastructure projects. *Journal of Infrastructure Systems*, 13(2):97–104, 2007.

E. Clément, D. Lamberton, and P. Protter. An analysis of a least squares regression method for american option pricing. *Finance and Stochastics*, 6(4):449–471, 2002.

M. Dailami, I. Lipkovich, and J. Van Dyck. Infrisk: a computer simulation approach to risk management in infrastructure project finance transactions. *World Bank Policy Research Working Paper*, (2083), 1999.

F. Dong and N. Chiara. Improving economic efficiency of public-private partnerships for infrastructure development by contractual flexibility analysis in a highly uncertain context. *The Journal of Structured Finance*, 16(1):87–99, 2010.

F. Dong, N. Chiara, N. Kokkaew, and J. Wu. Stochastic optimization of capital structure in privately funded infrastructure projects. *The Journal of Private Equity*, 15(1):36–47, 2011.

E. Engel, R. Fischer, and A. Galetovic. The basic public finance of public–private partnerships. *Journal of the European Economic Association*, 11(1):83–111, 2013.

A. L. L. Galera and A. S. Soliño. A real options approach for the valuation of highway concessions. *Transportation Science*, 44(3):416–427, 2010.

P. Glasserman. *Monte Carlo methods in financial engineering*, volume 53. Springer Science & Business Media, 2003.

M. B. Haugh and L. Kogan. Pricing american options: a duality approach. *Operations Research*, 52(2):258–270, 2004.

S. P. Ho and L. Y. Liu. An option pricing-based model for evaluating the financial viability of privatized infrastructure projects. *Construction Management & Economics*, 20(2):143–156, 2002.

T. Irwin. *Government guarantees: Allocating and valuing risk in privately financed infrastructure projects*. World Bank Publications, 2007.

K. Iyer and M. Sagheer. A real options based traffic risk mitigation model for build-operate-transfer highway projects in india. *Construction Management and Economics*, 29(8):771–779, 2011.

P. Jaillet, E. I. Ronn, and S. Tompaidis. Valuation of commodity-based swing options. *Management science*, 50(7):909–921, 2004.

F. A. Longstaff and E. S. Schwartz. Valuing american options by simulation: a simple least-squares approach. *Review of Financial studies*, 14(1):113–147, 2001.

N. Meinshausen and B. M. Hambly. Monte carlo methods for the valuation of multiple-exercise options. *Mathematical Finance*, 14(4):557–583, 2004.

M. Ranasinghe. Total project cost: a simplified model for decision makers. *Construction Management & Economics*, 14(6):497–505, 1996.

L. C. Rogers. Monte carlo valuation of american options. *Mathematical Finance*, 12(3):271–286, 2002.

J. Schoenmakers. The real multiple dual. *Available at SSRN 1465269*, 2009.

J. Schoenmakers. A pure martingale dual for multiple stopping. *Finance and Stochastics*, 16 (2):319–334, 2012.

J. N. Tsitsiklis and B. Van Roy. Regression methods for pricing complex american-style options. *Neural Networks, IEEE Transactions on*, 12(4):694–703, 2001.

X. Zhang. Financial viability analysis and capital structure optimization in privatized public infrastructure projects. *Journal of Construction Engineering and Management*, 131(6):656–668, 2005.